For Supervisor's use only

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93201Q

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2006
Statistics and Modelling

9.30 am Saturday 2 December 2006
Time allowed: Three hours
Total marks: 48

QUESTION BOOKLET

A 4-page booklet (S-STATF) containing Mathematical Formulae and Tables has been centre-stapled in the middle of this booklet. Before commencing, carefully detach the Formulae and Tables Booklet and check that none of its pages is blank.

Answer ALL questions.

Write ALL your answers in the Answer Booklet 93201A.

Show ALL working. Start each question on a NEW page. Number each question carefully.

Check that this Question Booklet 93201Q has pages 2–9 in the correct order, and that none of these pages is blank.

You are advised to spend approximately 30 minutes on each question.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

All questions are based on a product, strawberry yoghurt, produced by a company named Fastidious Foods. This company produces two types of strawberry yoghurt, light and standard, in 150 mL pottles. Each type of yoghurt is packaged in boxes containing six pottles.

QUESTION ONE (8 marks)

(a) The number of boxes of each type produced per day needs to satisfy the following constraints:

1. The total number of boxes produced is not to exceed 400, and there must be at least 200 boxes of standard yoghurt produced.
2. The number of boxes of standard yoghurt produced is to be at least twice the number of boxes of light yoghurt produced.
3. No more than seven boxes of standard yoghurt are to be produced for every two boxes of light yoghurt produced.

A box of light yoghurt yields a greater profit than a box of standard yoghurt.

Find the number of boxes of each type that should be produced per day in order to maximise the profit.

(b) Constraints 2 and 3 in (a) are changed so that the number of boxes of light yoghurt produced is between 33.3% and 40.0% of the number of boxes of standard yoghurt produced. Assume that constraint 1 in (a) still applies and that each box of light yoghurt still yields a greater profit than a box of standard yoghurt.

Find the number of boxes of each type of yoghurt that should be produced now to maximise the profit.

(c) A machine seals the pottles and another machine makes the boxes. On average, one in every 240 pottles has a faulty seal and one in every 50 boxes is faulty.

For a day's production of 400 boxes, find the expected number of boxes that would be faulty or would have a pottle with a faulty seal.

State any assumptions made.

QUESTION TWO (8 marks)

(a) The 150 mL pottles of standard yoghurt are filled on the production line. Fill volumes (in mL) are modelled by a normal distribution with a standard deviation of 8 mL.

(i) Calculate the mean fill volume so that only 1% of the pottles have a fill volume less than the labelled volume of 150 mL.

(ii) A random sample of 25 pottles yielded a mean fill volume of 155 mL.

Construct a 95% confidence interval for the mean fill volume. With reference to your answer to (i), what do you conclude? Justify your answer.

(iii) How many pottles are needed in a sample so that the sample mean fill volume is within 1.6 mL of the population mean, with 99% confidence?

(b) Two filling machines, A and B, are used independently to fill pottles of light yoghurt. It is suspected that the two machines are giving different mean fill volumes.

Using statistics, describe how you could check this.

QUESTION THREE (8 marks)

All incoming strawberries for making the yoghurt are checked for defects. An automatic scanner is used, followed by a visual inspection. The scanner removes 90% of defective strawberries and the visual inspection process removes 70% of any remaining defective strawberries. From previous records, the company knows that 4% of incoming strawberries are defective.

(a) Calculate the percentage of defective strawberries that are removed by the visual inspection process.

(b) Calculate the probability that a strawberry that was found to be defective had been removed by the scanner.

(c) A random sample of strawberries is taken before scanning.

What sample size is needed in order to have a 34% chance of finding at least two defective strawberries in the sample?

(d) A random sample of 40 strawberries taken before scanning had one defective strawberry.

Based on this result, is it likely that the percentage of defective incoming strawberries is now not 4%? Justify your answer.

QUESTION FOUR (8 marks)

The quality of strawberry yoghurt is scored on a scale of 0 (very poor) to 100 (excellent). Testing has shown that, after a particular period, the quality Q of the yoghurt starts to decrease, and that it may be modelled by the function:

$$Q(t) = \begin{cases} 100 \text{ for } 0 \leq t < D \\ Ae^{Bt} \text{ for } t \geq D \end{cases}$$

with $Q(D + 14) = 90$

where t is the number of days after manufacture
$Q(t)$ is the quality score at time t
D is the number of days from manufacture before the quality $Q(t)$ starts to decrease
and A and B are constants.

(a) Suppose that $D = 15$.

(i) Sketch a graph of $Q(t)$, for $0 \leq t \leq 30$.

(ii) Find the values of A and B.

(iii) A use-by date is calculated by finding out when the model indicates when the quality score will drop below 95.

Find the use-by date for yoghurt produced on 1st December 2006.

(b) Now suppose that D is normally distributed with a mean of 14.5 and a standard deviation of 1.2.

When $t = 20$, what percentage of strawberry yoghurt pottles would have a quality score below 95?

THE FOLLOWING INFORMATION IS USED IN QUESTIONS FIVE AND SIX.

Fastidious Foods wishes to investigate factors that have an impact on the sales of both types of yoghurt, so that they can make some sales forecasts. Data were collected over a period of three years, from December 2003 to November 2006.

You have been provided with the following data in Table 1:

The value of sales of standard yoghurt *SS* (in thousands of dollars, $000) for each month from December 2003 ($t = 1$) to November 2006 ($t = 36$) inclusive.

The 12-point centred moving mean *CMM* ($000) for sales of standard yoghurt for the months June 2004 ($t = 7$) to May 2006 ($t = 30$) inclusive.

The marketing costs *M* ($000) for each month from December 2003 to November 2006 inclusive.

The ratio of sales of standard yoghurt to the sales of light yoghurt $SR = SS : SL$ for each month from December 2003 to November 2006 inclusive.

In addition, you are provided with two graphs (page 8) as follows:

Figure 1: A graph of the value of monthly sales of standard yoghurt on which the *CMM* values have also been plotted.

A linear regression line fitted to the *CMM* points has equation $\boldsymbol{y = 89.068 - 0.9667t}$

Figure 2: A scatter plot of the ratio of sales of standard yoghurt to the sales of light yoghurt *SR* versus the marketing costs *M* for each month from December 2003 to November 2006.

A linear regression line fitted to 35 of the 36 plotted points (excluding the point with $SR = 1.8$) has equation $\boldsymbol{y = 1.8314 - 0.0646M}$

Table 1

Month	Months since Nov 2003 *t*	Sales of standard yoghurt *SS* ($000)	Centred moving mean *CMM* ($000)	Marketing costs *M* ($000)	Ratio *SR = SS*:*SL*
Dec 2003	1	110		5	1.55
Jan 2004	2	102		4.5	1.6
Feb 2004	3	97		9.5	1.3
Mar 2004	4	78		10	1.29
Apr 2004	5	81		10	1.15
May 2004	6	85		10	1.32
Jun 2004	7	39	80.5	11	1.09
Jul 2004	8	59	79.5	11.5	1.1
Aug 2004	9	67	78.5	12.5	1
Sep 2004	10	71	77.8	12.5	1.07
Oct 2004	11	88	77.3	8.5	1.4
Nov 2004	12	95	76.7	15	0.95
Dec 2004	13	98	77.6	11.5	1.06
Jan 2005	14	89	78.4	9.5	1.25
Feb 2005	15	88	77.5	9	1.17
Mar 2005	16	70	76.5	12.5	0.95
Apr 2005	17	75	75.1	13	0.83
May 2005	18	77	73.4	13	0.84
Jun 2005	19	69	71.9	15.5	0.99
Jul 2005	20	48	70.4	11	1.03
Aug 2005	21	56	68.8	13	0.95
Sep 2005	22	59	67.4	14.4	0.89
Oct 2005	23	66	66.1	16.5	0.83
Nov 2005	24	77	64.8	17	0.88
Dec 2005	25	79	63.7	15	0.91
Jan 2006	26	72	62.8	18.5	0.59
Feb 2006	27	68	61.8	16	0.79
Mar 2006	28	55	61.0	14	0.81
Apr 2006	29	59	60.7	14.5	1.8
May 2006	30	62	60.3	13	0.87
Jun 2006	31	58		13	0.91
Jul 2006	32	38		9	1.21
Aug 2006	33	41		18.5	0.69
Sep 2006	34	56		16	0.75
Oct 2006	35	61		18.5	0.72
Nov 2006	36	73		14	0.85

Figure 1: Standard Yoghurt Sales

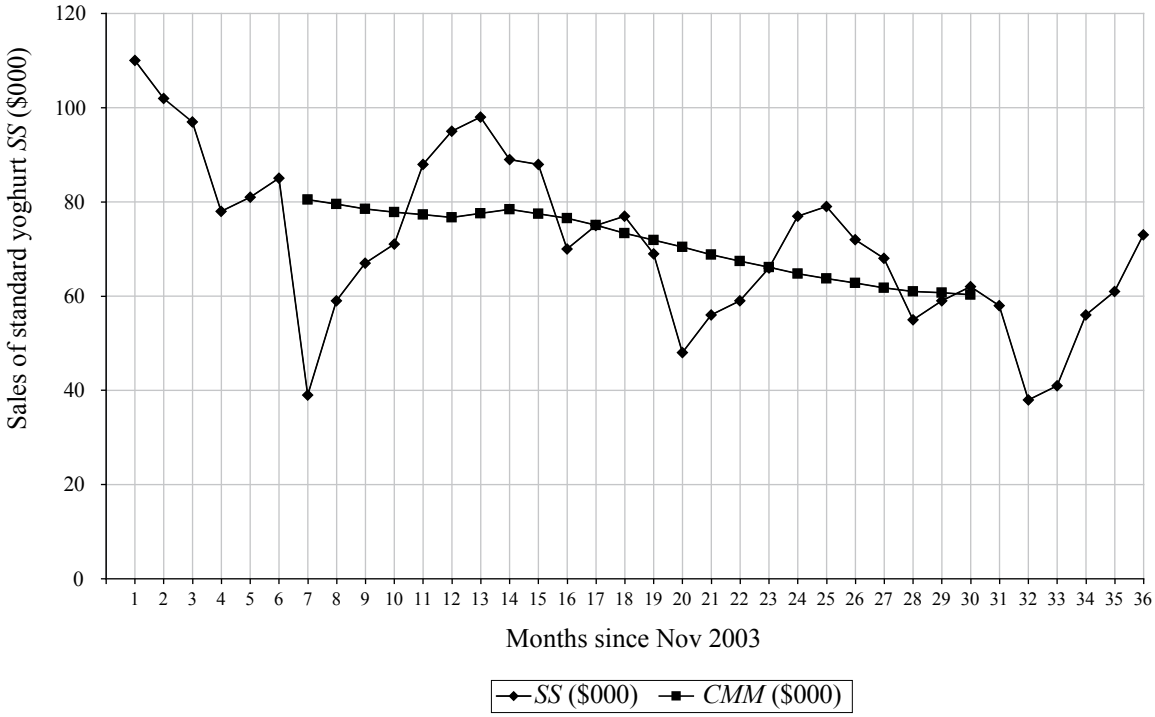
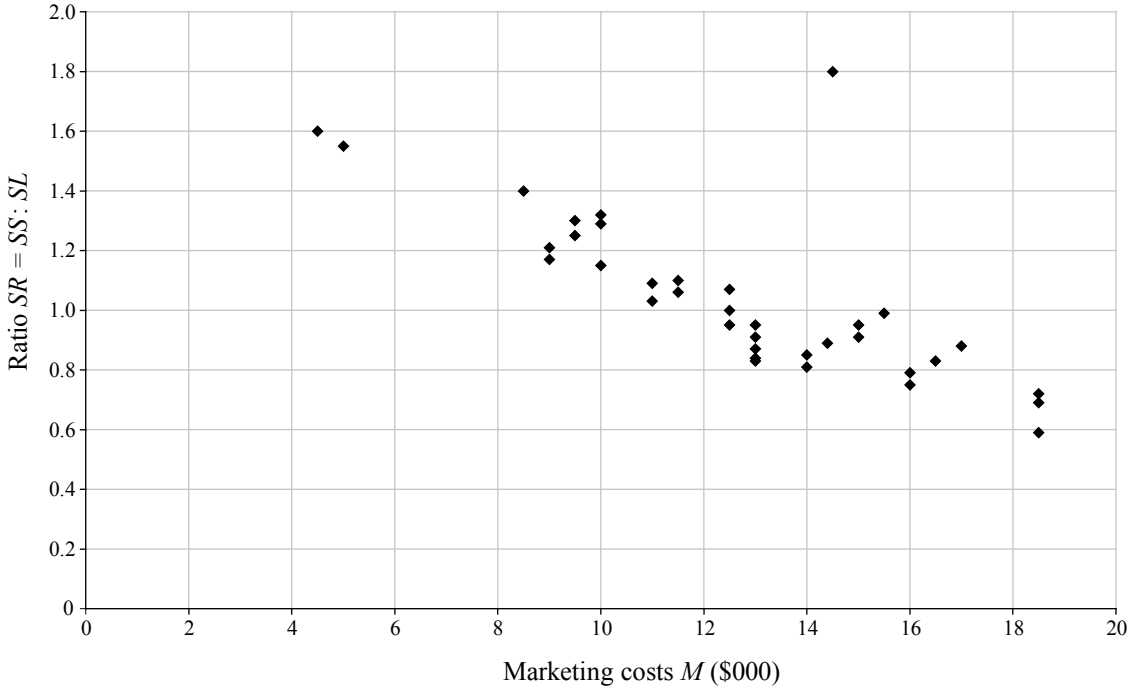


Figure 2: Yoghurt Sales Ratios and Marketing Costs



QUESTION FIVE (8 marks)

(a) Table 2 below shows the total yearly sales of standard yoghurt T (\$000), the price per pottle P (\$), and the number of sales outlets in operation N.

Table 2

Year	Total sales of standard yoghurt T (\$000)	Price per pottle P (\$)	Number of sales outlets N
Dec 2003 to Nov 2004	972	1.20	20
Dec 2004 to Nov 2005	872	1.28	24
Dec 2005 to Nov 2006	722	1.40	30

(i) The equation $T = a + bP + cN$ is used to represent this sales information.

Find a and b in terms of c.

(ii) Assume that $T = 0$ when $N = 0$. For the year from December 2006 to November 2007, also assume that the price per pottle will be \$1.45, and that 40 sales outlets will be in operation.

Obtain a prediction for the total sales of standard yoghurt for the year from December 2006 to November 2007.

(b) In February 2007 the company's marketing costs are budgetted to be \$25 000.

For February 2007 obtain a forecast for the:

(i) sales of standard yoghurt

(ii) sales of light yoghurt.

QUESTION SIX (8 marks)

By referring to the data in Table 1, the two graphs (Figures 1 and 2), and your calculated forecasts in Question 5, write a report of no more than two pages discussing sales of strawberry yoghurt. As part of your report, comment on the validity of your forecasts and suggest how this could be improved.